# Math-in-CTE

# Sample Automotive Lesson Plan

| **Lesson Title: Piston Displacement** | **Lesson AT07** |
|---|---|

| |
|---|
| Occupational Area: Automotive Technology |
| CTE Concept(s): Piston Displacement |
| Math Concept(s): Formula for finding the volume of a cylinder; computation |

| | |
|---|---|
| Lesson Objective: | Students will demonstrate a working knowledge of solving problems involving formulas with specified symbols (math concept) and its application in Automotive Technology: Engine Repair (technical field), while recognizing it in other contexts. |
| Supplies Needed: | AYES Curriculum Guide of math concepts:<br>Applied Academic & Workplace Skills for Automobile Technicians<br>Have various cylinders and old pistons from many different sizes of engines available. |

| THE "7 ELEMENTS" | TEACHER NOTES (and answer key) |
|---|---|
| **1. Introduce the CTE lesson.**<br><br>*Today we're going to talk about how to modify an engine to make the car go faster. What ideas do you have about how to make a car go faster?*<br><br>*Ask: What does "stroking" an engine mean?*<br><br>*Ask: What do you think the numbers on the sides of cars mean? Like a 426 Hemi Chrysler or a 427 on a Classic Chevy?*<br><br>*Ask: Why would you want to be able to figure out the cu in displacement of an engine?*<br><br>*Ask: So, how do you figure out the cubic inch displacement of an engine block? What engine dimensions or specifications determine the cubic inch displacement (CID)?* | The students will probably mention things like: change the headers, carburetion, cam shaft, raise the compression, stroke the engine…<br><br>Stroking the engine (ha, ha) means to change the crank shaft from short to long, therefore changing the cu. in. displacement of the engine.<br><br>The answer is that the numbers 427 stand for the cubic inch displacement of the engine. (427 cu. in.)<br><br>When you buy a car that needs to be rebuilt or restored and want to use it for drag racing, there is a size limit of 440 cu in. If you win and it's contested, your car can be taken apart and measured. If you are over 440 cu in, your car goes to the person contesting the race!<br><br>You would need to know the bore, which is the diameter of each cylinder, the stroke, which is the distance the piston travels from bottom dead center (BDC) to top dead center (TDC), and the number of cylinders in the engine block. The volume that the piston "sweeps" through is the cubic inch displacement of the engine. This volume can be represented as the volume of a right circular cylinder. |
| **2. Assess students' math awareness as it relates to the CTE lesson.**<br><br>Bring out many examples of cylinders and pistons, from diesel engines to minibikes. | Don't forget the rule: NRA…never refuse anything!<br>Begin collecting different sizes of pistons for demonstration purposes. |

| | |
|---|---|
| *Ask: What are the differences among all these cylinders?*<br><br>Ask:<br>1. What do we call the diameter of the cylinder?<br><br>2. What do we call the travel of the piston in the cylinder?<br><br>3. How do we determine piston displacement?<br><br>4. What would be a mathematical name for cylinder displacement?<br><br>5. How do we determine volume of a cylinder? (same as #4) | Students may say: they are small and big; they burn different fuels… You can judge the size of an engine by the size of the cylinder.<br><br>Listen to student discussion to see if they are familiar with vocabulary: Bore (diameter), stroke (height), cu in displacement, etc.<br><br>Answer (4): Total volume of the cylinders in the engine block<br><br>Engine displacement can be expressed in different units of measure (ie: cubic inches to liters to cubic centimeters).<br>• Define bore or diameter as the largest part across the cylinder.<br><br>• $V = \pi r^2 h \approx 3.14 r^2 h$ (volume of a cylinder)<br>$r = \frac{1}{2} d = \frac{d}{2}$ (relationship between radius and diameter)<br>$r^2 = \left(\frac{1}{2} d\right)^2 = \frac{d^2}{4}$<br>• Formula for piston displacement is based on the volume of a cylinder.<br>$PD = \frac{\pi d^2 SN}{4} \approx \frac{3.14 d^2 SN}{4} \approx 0.7854 d^2 SN$<br>where d is the cylinder bore; S is the stroke; and N is the number of cylinders.<br>• Bore – diameter of cylinder<br>• Stroke – height of cylinder<br>• PD – piston displacement |

| | |
|---|---|
| | • Volume of a cylinder:<br>$V = \pi r^2 h \approx 3.14\ r^2 h$<br>$\approx \frac{3.14\ d^2 h}{4} \approx 0.7854\ \ d^2 h$<br>where *r* = radius, *d* = diameter, and *h* = height of the cylinder<br>$Bore^2 * Stroke * 0.7854 * \#\ of\ \ Cylinders$<br><br>***NOTE: refer to conversion sheets and charts (Lesson #1)***<br>1 liter = 1000 cubic centimeters (cc)<br>1 cubic inch = 16.39 cc<br>cubic inch to cc ci X 16.39 cc<br>cc to cubic inch cc X .061 ci |
| **3. Work through the math example *embedded* in the CTE lesson.**<br>*Let's say we had a bore of 4 inches, a stroke of 3 inches…what would be the size of one cylinder?*<br><br>*What if you had a V8 engine? Multiply times 8.*<br><br>*So, if Mr. Ford needs a marketing strategy…what's he going to put on the side of his car:*<br>*301.59 or 302?*<br><br>*And in the 1960's, a 302 was really cool!*<br><br>*What if we convert it to metric? 302 becomes 5.0.*<br>*Which would you rather have on your car? 302 or 5.0?*<br><br>*Let's do some other examples:*<br><br>*Using a bore of 4.25 inches and a* | $(Bore)^2 * (Stroke) * \frac{\pi}{4}$<br>4 x 4 x 3 x .785 = 37.68 cu in (In one cylinder…)<br>V8 = 37.68 x 8 = 301.59 cu in<br><br>Use conversion of cu inch to liters…<br><br>PD = $0.7854(4.25)^2(3.76)(8)$ |

| | |
|---|---|
| *stroke of 3.76 inches in an 8-cylinder engine block should produce a volume or cubic inch displacement of approximately 427 cubic inches.*<br><br>*Another problem:*<br><br>*What is the cubic inch displacement of an 8-cylinder engine having a bore of 4.094 inches and a stroke of 3.76 inches? The result should be approximately 396 cubic inches, which is the CID of engine blocks used for Chevy Chevelles back in the middle to late 1960's. The number 396 is also located on the side front fenders of these cars.* | PD = 426.7 ≈ 427 cu in<br><br>≈ means approx. equal to<br><br>$piston\,displacement = \frac{\pi (bore)^2 (stroke)(number\,of\ \ cylinders)}{4}$<br><br>$= .7854\, d^2 SN$<br><br>where: *d* = bore<br><br>*S* = stroke<br><br>*N* = number of cylinders<br><br>$\pi$ = Pi (3.1416…) |
| **4. Work through *related, contextual* math-in-CTE examples.**<br><br>1. Find the piston displacement for a 4-cylinder engine with a 3 3/4" bore and a 3 1/2" stroke.<br><br>2. Find the increase in PD if a 8-cylinder engine with a 3.5" bore and a 3.75"crankshaft stroke is increased to a 3.55" bore. | 1. PD = .7854 $(3.75)^2$(3.5)(4)<br>PD = 154.6 ≈ 155 cu in<br><br>2. $PD_1$ = .7854 $(3.5)^2$(3.75)(8) = 288.6 cu in<br>$PD_2$ = .7854 $(3.55)^2$(3.75)(8) = 296.9 cu in<br>Increase difference = 8.3 cu in<br>Have students actually measure a bore and a stroke and figure out cubic inch displacement convert to cc and liters. |

| | |
|---|---|
| **5. Work through *traditional math* examples.**<br>3. Find the volume of a soup can with the diameter of 2 7/8" and the height of 4 3/8".<br>4. Find the height of a cylinder with a volume of 27 cubic inches and a radius of 2 inches. | 3. $V = \frac{\pi(2\frac{7}{8})^2(4\frac{3}{8})}{4}$ =28.4 cu in<br>4. $27 = \pi (2)^2 h$<br>$27 = 12.57h$<br>$h = 2.15$ in |
| **6. Students demonstrate their understanding.**<br>*Ask: So what advice would you give the student that has to restore/rebuild the 427 engine to compete in a drag race?* | Begin with a class discussion of advice to give the student. Continue by asking students to write out a list of steps the student should follow to rebuild the engine.<br><br>Students could also choose a different example engine and describe to the class how they would make the engine drag race ready.<br><br>Students could use the attached worksheet for homework, to be filed in student portfolio. |
| **7. Formal assessment.**<br>Possible Test Questions<br>1. Find the piston displacement for a 4-cylinder engine with a 3" bore and a 2 1/2" stroke.<br>2. Find the volume of a cylinder with a radius of 10 inches and a height of 20 inches. | 1. PD = .7854 $(3)^2(2.5)(4) = 70.69 \approx$ 71 cu in<br>2. $V = \pi r^2 h$<br>$V = 3.14\ (10)^2\ (20)$<br>V = 6283.19 cu in |

# Practice with Piston Displacement Worksheet

Name_____ Session_____ Date_____

1. Find the piston displacement for a 6-cylinder engine with a 8.56 cm bore and a 6.9 cm stroke.

2. Find the piston displacement for an 8-cylinder engine with a 4.13” bore and a 4” stroke.

3. Find the volume of water in an above ground, circular swimming pool that is 3 feet high and 15 feet across.

4. What is the radius of the pool mentioned in problem 3?

5. What is the volume of the pool if the radius is 5 feet and the height is 5 feet?

6. Find the height of the silo if the volume is 14,322 square feet and the diameter is 20 feet.

7. Find the volume of a cylinder with a diameter of 36 cubic inches and a height of 6 inches.

8. Find the height of a cylinder with a volume of 28 cubic inches and a radius of 2.5 inches.

# Piston Displacement Worksheet Key

1. PD = $.7854(8.56)^2(6.9)(6)$
   PD = 2382.53 cc

2. PD = $.7854(4.13)^2(4)(8)$
   PD = 428.69 cu in

3. $V = \pi r^2 h$
   $V = \pi\ (15/2)^2(3)$
   V = 70.69 cu ft

4. r = 7.5 ft

5. $V = \pi r^2(h)$
   $V = \pi\ (5)^2\ (5)$
   V = 392.7 cu ft

6. $14322 = \pi(20/2)^2 h$
   $14322 = 3.14\ (10)^2\ h$
   14322 = 314.16h
   14322/314.6 = h
   h = 45.52 ft

7. $V = \pi\ r^2 h$
   $V = 3.14\ (36/2)^2\ 6$
   V = 6107.3 cu in

8. $28 = \pi\ (2.5)^2 h$
   28 = 3.14 (6.25) h
   28 = 19.63h
   28/19.63 = h
   h = 1.49 in